

DOCUMENT RESUME

ED 322 134

SP 032 536

AUTHOR Schram, Pamela; And Others
TITLE Thinking about Teaching Subtraction with Regrouping:
A Comparison of Beginning and Experienced Teachers'
Responses to Textbooks. Research Report 89-5.
INSTITUTION National Center for Research on Teacher Education,
East Lansing, MI.
SPONS AGENCY Office of Educational Research and Improvement (ED),
Washington, DC.
PUB DATE Mar 90
NOTE 29p.
AVAILABLE FROM National Center for Research on Teacher Education,
116 Erickson Hall, College of Education, Michigan
State University, East Lansing, MI 48824-1034
(\$4.75).
PUB TYPE Reports - Research/Technical (143)
EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS Beginning Teachers; Elementary Education; Elementary
School Mathematics; Elementary School Teachers;
*Manipulative Materials; *Mathematics Instruction;
*Number Concepts; Place Value; *Subtraction;
*Teaching Experience; Teaching Methods; Textbook
Evaluation; *Textbook Selection

ABSTRACT

This paper reports a comparative analysis of beginning and experienced teachers' thinking about teaching subtraction with regrouping commonly called "borrowing"), and the role that textbooks play in their deliberations. Twelve beginning and nine experienced teachers appraised and compared two contrasting textbook selections dealing with this topic and described how they would teach it with or without either of the textbooks. The analysis revealed both differences and similarities between the experienced and beginning teachers. Not surprisingly, experienced teachers had more elaborated lenses for looking at the textbooks and thinking about how to teach the topic. While both experienced and beginning teachers seemed to assume that manipulative materials were inherently worthwhile, neither beginning nor experienced teachers discriminated among particular concrete materials and all seemed to think that seeing or touching such materials automatically produced understanding. Nor did the two groups differ significantly in their understandings of multidigit subtraction. This finding challenges the common belief that teachers learn about their subjects by teaching them. Rather, subject-specific knowledge of children's learning seems a more likely outcome of teaching experience. An important role for textbooks appears to lie in developing teachers' understandings of the appropriate use of concrete materials. (Author/JD)

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Thinking About Teaching Subtraction With Regrouping: A Comparison of Beginning and Experienced Teachers' Responses to Textbooks

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**THINKING ABOUT TEACHING SUBTRACTION WITH REGROUPING:
A COMPARISON OF BEGINNING AND EXPERIENCED TEACHERS'
RESPONSES TO TEXTBOOKS**

**Pamela Schram, Sharon Feiman-Nemser,
and Deborah Loewenberg Ball**

Published by

The National Center for Research on Teacher Education
116 Erickson Hall
Michigan State University
East Lansing, Michigan 48824-1034

March 1990

This work is sponsored in part by the National Center for Research on Teacher Education, College of Education, Michigan State University. The National Center for Research on Teacher Education is funded primarily by the Office of Educational Research and Improvement, United States Department of Education. The opinions expressed in this paper do not necessarily represent the position, policy, or endorsement of the Office or the Department.

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Abstract

This paper reports a comparative analysis of beginning and experienced teachers' thinking about teaching subtraction with regrouping and the role that textbooks play in their deliberations. The teachers were asked to compare and appraise two contrasting textbook selections dealing with this topic and to describe how they would teach it. The analysis revealed both differences and similarities between the experienced and beginning teachers. Not surprisingly, experienced teachers had more elaborated lenses for looking at the textbooks and thinking about how to teach the topic. While both experienced and beginning teachers seemed to assume that manipulative materials were inherently worthwhile, neither beginning nor experienced teachers discriminated among particular concrete materials and all seemed to think that seeing or touching such materials automatically produces understanding. Nor did the two groups differ significantly in their understandings of multidigit subtraction. This finding challenges the common belief that teachers learn about their subjects by teaching them and suggests an important role for textbooks in developing teachers' understandings of the appropriate use of concrete materials.

THINKING ABOUT TEACHING SUBTRACTION WITH REGROUPING: A COMPARISON OF BEGINNING AND EXPERIENCED TEACHERS' RESPONSES TO TEXTBOOKS¹

Pamela Schram, Sharon Feiman-Nemser, and Deborah Loewenberg Ball²

Because textbooks dominate mathematics instruction in elementary schools (Fey, 1978; Goodlad, 1984; Schulle et al., 1983; Stodolsky, 1985), understanding how teachers respond to textbooks when they are thinking about teaching a particular topic is important (Ball and Feiman-Nemser, 1988). What do teachers consider when looking at textbooks? How do they decide whether and how to use particular sections? How are their deliberations influenced by their knowledge and assumptions--about the content, about teaching the content, and about what helps kids to learn it? What role does teaching experience play?

Methodology

This paper reports a comparative analysis of beginning and experienced teachers' thinking about teaching a particular topic--subtracting multidigit numbers--and the role that textbooks play in their deliberations. The interview data on which the analysis is based come from a national longitudinal study of teacher education and learning to teach currently underway at the National Center for Research on Teacher Education (NCRTE) at Michigan State University. The study examines what teachers are taught and what they learn in 11 diverse preservice, induction, inservice, and alternate route programs around the country. It combines case studies of programs with longitudinal studies of participants' learning (see NCRTE, 1988).

Study Participants and Data Collection

In this paper we analyze and compare the responses of 12 beginning and 9 experienced elementary teachers. The beginning teachers were all in their first year of teaching. The experienced teachers had taught for an average of 11 years. Although these data come from a longitudinal study of teacher learning in teacher education programs, this analysis draws on the baseline data. Thus it reflects these teachers' thinking prior to their

¹ This paper was presented at the annual meeting of the American Educational Research Association in San Francisco in March 1989.

² Pamela Schram, an instructor in teacher education at Michigan State University, is a research assistant with the National Center for Research on Teacher Education. Sharon Feiman-Nemser, a professor of teacher education at MSU, is an NCRTE senior researcher. Deborah Ball, an assistant professor of teacher education at MSU, is an NCRTE senior researcher. The authors gratefully acknowledge the assistance of other members of the research staff of the National Center for Research on Teacher Education, especially Barbara Camilleri, Perry Lanier, James Mead, Michelle Parker, and Richard Prawat.

participation in the programs we were studying.³ Participants were presented with two contrasting textbook sections, asked to appraise each selection and to compare the two. They were also asked to describe how they would teach this content (subtraction with regrouping), with or without either of these textbooks.⁴

The Textbook Sections

Although mathematics educators refer to the topic treated in the selections as "subtraction with regrouping," it is commonly called "borrowing" after the standard algorithm that everyone learns in elementary school for subtracting multidigit numbers, for example:

$$\begin{array}{r} 54 \\ - 46 \\ \hline \end{array}$$

The procedure is rooted in concepts of place value. In the problem above, one "regroups" the tens and ones of 64 to form 5 tens and 14 ones (which still equals 64). Then one subtracts 14-6, and 50-40. Usually people do not think explicitly about this, but perform the operation mechanically and automatically with the numbers. Able to compute accurately, they may have only a tacit understanding of the underlying concepts. For example, in explaining the procedure, they may say something like, "You can't take 6 away from 4, so you borrow from the 6; it becomes a 5, put a 1 next to the 4, then you can do it: 14-6 = 8, 5-4 = 1, so it's 18."

Two second-grade textbook sections dealing with two-digit subtraction with regrouping were used for this task (*Mathematics Around Us*, Scott Foresman, 1978; *Real Math*, Open Court, 1985). Each section represents one day's lesson and comes at a parallel point in the book's development of the topic. Participants were presented with copies of the student pages and the teacher's guide pages and asked how they would teach this topic, what they thought of each of the textbooks, and whether and how they would use them in teaching the topic.

The two texts were selected because they present interesting contrasts in their focus, format, and representation (copies of the text sections are appended to this paper). The first (Appendix A) focuses on identifying the tens and ones place and on carrying out the

³ The beginning teachers had all graduated from the same preservice teacher education program, a program with some unifying themes. Thus they do not represent a random sample of beginning teachers. Similarities in their knowledge, assumptions, or ways of thinking may be a consequence of the similarity in their preservice preparation. None of them, however, had begun to teach nor to participate in the induction program we are studying.

⁴ This task was part of a longer interview in which participants were asked to respond to various scenarios built around tasks related to teaching. The purpose of the interviews was to learn about their knowledge and beliefs—about mathematics and writing (the focal subjects of this study), as well as about learners, learning, and contexts of teaching—and the ways in which they drew on these different kinds of knowledge in thinking about teaching.

procedure correctly instead of on *regrouping* of numbers. Examples are formatted to help students attend to the placement of numbers:

tens	ones
6	4
-4	6

The second (Appendix B), which takes a more conceptual approach, has illustrations of bundles of sticks and loose sticks to model tens and ones and their regrouping of tens and ones.

The first section is colorful and sets the exercises in the context of a story about what was sold at a fair. With a picture of a single object such as a glass of Coke, a ball, or a hot dog above each problem, students are supposed to consider the question, "How many were not sold?" The second section is black and white and the practice page is unadorned. Yet another difference lies in the nature of student practice: With one exception, all exercises presented in the first section require regrouping; the second provides mixed (and novel) practice, including some problems that require regrouping and some that do not, and even one that goes beyond what has been taught (100-26).

We selected the textbooks because of these differences. We thought, for example, that the colorfulness and the real-life examples in the first section might appeal to teachers concerned with making mathematics fun or relevant, while the emphasis on concrete or pictorial representation of the underlying meanings would appeal to those concerned with meanings or the inherent value of using manipulatives. We also were curious about whether teachers would consider the nature of the student practice examples.

Data Analysis

This analysis compares the beginning and experienced teachers' ideas about teaching subtraction with regrouping and their responses to the textbook sections. The analysis focuses on their knowledge and assumptions about the topic, about teaching the topic, and about student learning. Each teacher's interview transcript was read and all considerations mentioned were entered on an analytic table constructed to reflect the range of issues represented. These issues derived from a conceptual analyses of the text materials, the content itself, ideas about how children learn, and on unanticipated considerations introduced by the teachers. Tabulations of the content of individual teachers' considerations were then compared across all participants and between the two groups (experienced and beginning teachers).

How Teachers Construed the Content

In comparing the textbooks, describing what students would have to know to learn "this," and explaining how they would teach "it," beginning and experienced teachers revealed quite different understandings of the mathematical content. Their ideas ranged from borrowing (e.g., go to the next column and "get" what you need) to ideas about the significance of place value (e.g., need to understand "tens" and "ones") to talk about regrouping (e.g., "bundling" of tens) to the notion of equivalent exchanges (e.g., trading 1 ten for 10 ones and maintaining equivalent value). Beginning and experienced teachers fell along the entire continuum with the beginning teachers emphasizing borrowing and place-value understanding.

Teachers who thought these two textbook pages were about "borrowing" focused their explanations on the traditional algorithmic procedure. They talked about borrowing as one might go to a neighbor's house to "borrow" a cup of sugar:

Brady:⁵ But if you do not have enough ones, you go over to your friend here who has plenty and so you go through this idea of borrowing from the neighbor or crossing off.

Fay: You can't subtract a bigger number from a smaller number. . . . You must borrow from the next column because the next column *has more in it*.

One teacher struggled to explain the notion of "borrowing" and finally said, "It's kind of weird to me, too. Like you're taking one ten over here and now all of a sudden . . . it's a 14." The teachers who talked about "borrowing" seemed comfortable about their own ability to *do* subtraction with regrouping but were less able to articulate the conceptual underpinnings of this procedure.

While most of the experienced teachers talked in terms of place value and regrouping, the responses in both groups ranged from beginning notions of regrouping to more developed or elaborated ones:

Beverly: You would set it up and how you would regroup and take one of the bundles of ten and take it over to the ones.

⁵All names of students are pseudonyms. Beginning teachers names start with the letter "F" and experienced teachers names start with the letter "B."

Bernice: I would start with what the number 64 means. They have to know that six is really 60 and the four is only 4 ones and then when you do the regrouping part that you go over and you take one of the sixes, which is really 6 tens. The wording of it, I think, is extremely important. You have got to say, you are going to use--and, of course, I usually do it visually--one pack of the tens. One section of the 6 tens and now that will be 5 tens or 50, now where does the ten go? You cannot let it go, it is going to go over and help the ones. Now the four becomes what? 14. And then I would show that the number 64 and the number 5 tens and 14 ones equal the 64. I would try to, draw the comparison between that because when you are doing the regrouping it is not so much knowing the facts [subtraction number facts], it is the regrouping part that has to be understood.

The clearest and most developed explanation of regrouping and equivalent exchanges came from a beginning teacher:

Faith: They have to understand how exchanges are done . . . with the base ten blocks when you reach a certain number--ten, in base ten, in the ones column that is the same as saying 10 ones or 1 ten . . . they have to get used to the idea that *exchanges are made within place values and that it does not alter the value of the number [Italics added]*. . . . Nothing happens to the actual value, but exchanges can be made.

Teachers' Responses to the Textbooks

Given these perceptions of the content or topic, how did teachers respond to the textbook selections? What did they like and dislike about the excerpts? Did they prefer one over the other? How, if at all, would they use this material in their teaching? In analyzing teachers' responses to the two textbook selections, three points stand out. First, both groups commented on the same features and expressed the same range of opinions. Second, beginning teachers expressed a clear preference but there was no pattern of preference among the experienced teachers. And third, experienced teachers talked about the selections in terms of their own teaching experience, making it difficult to separate talk about texts from talk about teaching.

Overall, the beginning and experienced teachers commented on similar features and expressed the same range of opinions about them. Even popular features had their critics in both groups. Most of the beginning teachers liked the format of the problems in the first selection. They thought having a line between the columns and labeling them "ones" and "tens" would be helpful reminders to students. At the same time, most found the illustrations confusing because they did not portray the problem accurately. As Felix put it,

The other thing I don't like about it is that, even though they give you solid objects to think about that are of particular interest to kids, like fruit, toys, money . . . they only symbolize them in an abstract way which really doesn't tie into the problem. In other words, there aren't 64 sodas or 46 gone. There is just one. . . . I think they should use smaller numbers so that they could conveniently put symbols of whatever. . . . If they wanted to put 91 hot dogs, make it 19.

While a few thought the strategy of creating a story about the problems made the math more "relevant" or "interesting," others felt that figuring out "how much food was not sold" would be meaningless unless students had actually participated in a school fair.

Almost all the beginning teachers favored the second textbook because of the illustrations. Comparing the two selections, one beginning teacher said that the "visuals" in the first textbook were "just symbols," while the visuals in the second "tell me about the relationship with the numbers." Some saw the pictures as a model of the idea of place value ("showing how many there are"--5 sets of ten and 3 ones); others saw the pictures as demonstrating the requisite procedure ("showing how you are subtracting them out"). Most said they would use "real sticks" in combination with or instead of the pictures.

At the same time, several beginning teachers commented that the format of the problems in the second selection seemed confusing and they wondered why the text said "5 tens and three" instead of "3 ones." Only one expressed appreciation for the plain problems on the practice page, written out as one would encounter them in "real-life situations."

Francesca: If you go to the grocery store and they sell whatever for 95¢, they are not going to tell you 9 tens and 5 ones. You are going to see it in this form [95].

Finally, several commented on the usefulness of the teacher's guide with its specific, step-by-step suggestions.

Experienced teachers said many of the same things about the two selections. Regarding the first excerpt, three liked the idea of labeling the tens and ones column and separating them with a line; two said the illustrations would be distracting. Unlike the beginning teachers, however, no experienced teacher praised the strategy of setting problems in the

context of a story about a school fair. One said she liked the idea of having something the kids could relate to but worried about the "distracting factor." Another remarked that the whole idea seemed pointless. Talking about the second selection, experienced teachers said the pictures offered "more visual clues," the teachers' guide contained better "teaching tips," but the problem format was "confusing."

Most striking was the tendency of experienced teachers to look at the selections through the lens of their own teaching experience. This enabled them to consider how students would respond to the material and what they would do with it. While beginning teachers gave a blanket endorsement of the problem format in the first selection, for example, experienced teachers saw its usefulness in terms of particular students. One said: "I think having it split up into tens and ones would be good for somebody who is just beginning, for younger kids who are truly having trouble." Commenting on the second selection, another remarked: "This would be easy for students who get the concept of ones and tens and regrouping, but it would be harder for those that did not." Often we had difficulty separating out experienced teachers' talk about the text from their talk about teaching because the two were so intertwined.

Those teachers who already had a "curriculum script" (Putnam, 1987) for teaching subtraction with regrouping immediately fit the texts into that sequence. Barry, one of the experienced teachers described his strategy: "First you use manipulatives to 'show' an example or two so the students can understand the 'why,' then you drill them more like you normally see it without manipulatives." Since, in this teacher's view, the second text was more conceptual and the first, more of a "drill-type thing," he said he would begin with the second and move to the first:

I prefer both but I would start with two. I would do that first. I might spend a lesson on it with two or three examples to get the concept across of what is happening when you rename, when you borrow ten out of a bundle and make them ones. And then see if they can transfer that idea to a regular example where it still has the tens and ones written just slightly differently but, can you do it now without the manipulatives?

While most beginning teachers preferred the second selection because of the "visuals," there was no pattern of preference among the experienced teachers. Of the nine, four preferred the second selection, three said they would use both, and two favored the first selection, even though they would still use manipulatives. One experienced teacher did not like the way the problems were written out in the second textbook: "I think it is much too difficult. It is too spread out. . . . Numbers aren't written this way." While she rejected the more conceptual textbook, she still said she would use manipulatives to teach the content of regrouping.

Teachers' Thinking About Teaching the Topic

When teachers talked about teaching the content, differences between the beginning and experienced teachers stood out. Unlike the beginning teachers, the experienced teachers seemed to recognize the topic as part of a larger sequence of topics. They also presented more elaborate plans for how they would teach it. Beginning teachers were less sure about what came before and where their instruction should begin. When talking about how they would teach the topic, they tended to describe activities and materials rather than present an instructional sequence.

Most of the experienced teachers could locate the topic in the context of a mathematics curriculum. They seemed to have some idea of what came before ("I would probably have already taught them about groups of tens"), what they should focus on ("It is the regrouping part that has to be understood right from the start"), and what should come next. One teacher shared an analogy that she uses to explain subtraction: "If you have three eggs in a basket, can you take eight out of that basket?" Another explained what is confusing to students about the topic:

Brady: Probably the confusion comes in when they see these numbers. They want to do the tens first and then the ones because of the way we read. I think that has something to do with it--the fact that we read from left to right.

In contrast, many of the beginning teachers had trouble figuring out what students would already know and where their instruction should begin. Instead of focusing on subtraction with regrouping, some talked mostly about simple subtraction or place value. It seemed as though they did not recognize the topic as significant in its own right within a longer developmental sequence. For example Francine said,

I would start with the simple numbers like one digit numbers and when I got into the two digits, I would first do ones that you would not need to borrow from."

Beginning teachers also expressed more uncertainty about how to teach the content. One admitted: "I do not know too much about teaching." Another said: "I'm not sure about the levels yet. Maybe it would be hard for some second graders." In general, they did not display the kind of pedagogical knowledge of learners and content that some experienced teachers revealed in their responses.

Overall, the teachers' plans for teaching seemed to fall along a continuum. At one end were plans consisting mostly of activities and materials but no clear instructional goal. At

the other end were plans that seemed to fit Putnam's (1987) definition of a "curriculum script," an "ordered set of goals and actions for teaching a particular topic." In between were plans with vague goals and instructional sequences.

Beginning teachers tended to talk in terms of activities and materials rather than curriculum scripts, but there were also experienced teachers who fit this pattern. Fleur and Brady's comments represent each group:

Fleur: I'd use the bundles and have them come up with a number they wanted to subtract. They could think of a situation . . . say if we were having a fair, what might we sell? I'd have them come up with a problem and show them with the sticks.

Brady: I have used this idea of a bunch of tens to make one and then opening it up to show that it is really worth ten. So with second grade, definitely a lot of visuals and stuff to touch and a lot of examples over and over again and sending the little kids to the board.

A common script that both beginning and experienced teachers described involved starting with manipulatives, gradually adding work with symbols, and eventually eliminating manipulatives in favor of pencil-and-paper problems. This sequence allowed teachers to use both textbook selections, beginning with the second and moving to the first. Francine, a beginning teacher, illustrates this notion:

I would start with the manipulatives. We would spend a lot of time with that in group work and in individual conferences. Then bit by bit, I would start introducing the symbols, the actual formula-- $53 \text{ minus } 25 \text{ equals}$. . . I would make sure they saw a clear connection between what they were doing on their tables with these manipulatives and what we were doing at the board. . . By the end of the year, I would try to wean them off the manipulatives and more toward the symbols.

While experienced teachers tended to have more developed teaching ideas, the clearest curriculum script came from a beginning teacher, Faith, who said she would "come up with a good questioning strategy so they discover it on their own":

What I would do is show how each one of these bundles is ten, is 1 ten or 10 ones. I would make sure that was clear. . . And then I would show that now you have 1, 2, 3, 4 tens and 13 ones and then subtract in that fashion. . . I would say to the child, so you are telling me that we have not added anything or subtracted anything

to the 53, right? . . . I would rely heavily on the manipulatives and then proceed to the more abstract.

Using Manipulatives

Teachers' Thinking About Manipulatives

A prominent theme in teachers' talk concerned the use of manipulatives. Regardless of which textbook they preferred, all the teachers said they would use concrete materials to teach this content. They identified a wide assortment of materials; however, they were not always discriminating in their selection. The use of manipulatives was widely assumed to lead to student learning.

Some teachers seemed to have a generic view of manipulatives. They valued such materials simply because they provided a "hands-on" experience or made mathematics more "relevant" or "interesting."

Belinda: I would have them have their own pencils or sticks . . . take out *some* manipulative . . . little chips or little buttons, anything that you have got.

Frances: I could have them use a manipulative that would stand for the ice cream float . . . you know, the cubes or the Cuisenaire rods . . . popsicle sticks, beans . . .

Felice: . . . a more concrete way of learning it . . . it makes more sense in your mind when you can touch something and move it around.

Barry: Another good idea might be coins, using money because kids like money. They relate to money . . . maybe a different type of manipulative. . . . But it has got to be something that really relates to their life, something that fits in with them . . . children will learn best when they feel they need to learn it.

Teachers' views of learning were reflected in their comments about using manipulatives to help students move from "the concrete to the abstract." Over and over, beginning teachers said that manipulatives help students "see" mathematical ideas, as though seeing would automatically produce understanding. It was not always clear what ideas students were supposed to understand or how they would get from "seeing" to understanding. Florence, a beginning teacher, illustrates this idea:

As far as the borrowing or the renaming goes I think it is much easier for a child to get the concept of that if maybe you have blocks or cubes. . . . Concretely seeing it before you do it on paper. I think that would help with keeping the numbers straight and all the problems that kids have when they're first starting to do problems on paper.

Experienced teachers also talked about manipulatives as an aid to understanding; however, they tended to be clearer about what they wanted to accomplish or why a particular material would be helpful. Most often, they wanted students to "visualize" the subtraction algorithm in order to understand why they performed the various steps. This fits with their tendency to select materials that could be taken apart much like the bundles of sticks in the second textbook selection. Mostly they focused on the concept of grouping, talking about using "bunches of tens," "literally break apart one group of tens," "using bundles because you can take them apart." Bridget explained:

It seems as though actually seeing the bundles of tens and the threes and maybe even having them for the kids to mimic that picture [in the text] and then to do that would be good. . . . I think using bundles because you can take them apart and showing them exactly what they are doing [in the algorithm] and doing it on paper at the same time or doing it on the board at the same time. . . . I think to actually see it would make it easier.

Only two teachers, both beginners, talked explicitly about the notion of trading as an equivalent exchange and mentioned materials that could help students grasp this idea. Faith's comments illustrate this concept well.

They have to discover the relationship between the ones place and the tens place by actually manipulating things that can teach them that. I do not think I can stand up and say that 6 tens is the same as 60 ones. I think they have to discover it whether using a counter or working with base ten where they have to keep exchanging. They have to get used to the idea that exchanges are made within place value and that it does not alter the value of the number.

While teachers appreciated the role of manipulatives as an aid to understanding, they seemed to view them as crutches rather than tools, something to use on a temporary basis and get rid of as soon as possible. They talked about using manipulatives "in the beginning" or doing "an example or two" or "a lesson" to demonstrate how the manipulative "matched" the algorithm, but their goal was to "wean" students from reliance on such materials and to get on to the main goal of operating exclusively at the symbolical level. Furthermore, they

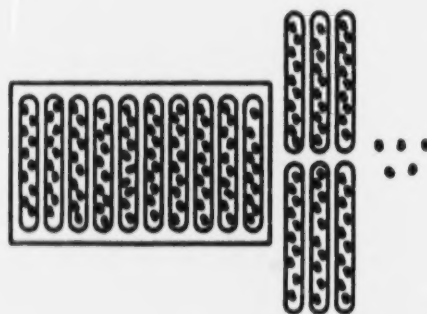
seemed to believe that operating with the symbolic forms was not only preferable but also a sign of understanding. Statements by two experienced teachers illustrate this idea:

Barry: I might spend a lesson on it with two or three examples of it to get across what is happening when you rename . . . then see if they can transfer that idea to a regular example . . . without the manipulatives. . . . I would not continue too much with the manipulatives as long as they have had a lesson or two . . . they can see that I have shown them what is really going on.

Bernice: OK, At the very beginning . . . , I would have bunches of, literally bunches of the tens . . . literally break apart one group of tens, break it away from the tens to give it to the four to make it 14. . . . I have already taught them about groups of tens. I use the chalk at a side angle and I just make a large blur and they know that is ten inside. . . . I would move it around on the board or use the overhead projector and have it more or less travel over to the four because it really is part of it.

Thoughtful Use of Manipulatives

Many different manipulatives could provide useful models for understanding subtraction with regrouping. To choose appropriately, however, teachers have to understand the benefits and limitations of particular materials. For example, some materials are structured to highlight the relationship between hundreds, tens, and ones in our numeration system. To help students understand this concept, the representations show hundred as *one* of something that clearly contains *10* tens. For example, if 10 pinto beans glued on a stick represents *10*, then 10 of those sticks glued together would represent both the *one* and the idea of *hundred*. That is, the object formed with the ten sticks is a *one* hundred-tile, made out of 10 ten-sticks. Each ten-stick is made from 10 individual beans.



¹²17

Multibase arithmetic blocks (also referred to as base ten or Dienes blocks) are another example of a composition model.



Other materials are useful in demonstrating grouping. Loose and bundled ice cream sticks are valuable as representations of tens and ones because students can actually assemble and disassemble the bundles and still see all 10 objects in a bundle.



A third group of manipulatives requires the learner to know particular attributes about the material in order to understand what they represent. For example, you cannot look at a dime and "see" that it represents 10 pennies. The same applies to chip trading games where chips of different colors represent different values. The materials themselves do not directly represent the attributes of the concept.

Even with appropriate materials, student understanding is not automatic; the connections between the manipulatives and the mathematical ideas must be made explicit (see, for example, Driscoll, 1981; Hiebert, 1984; Resnick, 1982). Teachers play an important role in establishing explicit links between concrete materials and symbolic representations.

If concrete materials are going to be useful, frequent, explicit links must be made between the physical and symbolic representations. . . . It is not just the use of concrete materials that improves mathematical understanding, but rather the explicit construction of links between understood actions on the objects and related symbol procedures. (Hiebert, 1984, p. 509)

Moreover, students need time to feel comfortable with materials and to observe and talk about the mathematical features they represent. In general, teachers tend to rush students from manipulatives to symbols (Reys, Suydam, and Lindquist, 1984).

Conclusions

These beginning and experienced teachers' ideas about teaching subtraction with regrouping highlight the interaction between teachers' knowledge and assumptions--about content, about teaching the content, and about what helps students to learn the content--and their responses to textbooks. One might expect to see substantial differences between beginning and more experienced teachers in terms of their thinking about teaching subtraction and in terms of their responses to textbook sections about the topic. While this may be true in some areas, it does not hold across the board. In this concluding section, we discuss one area where we noticed differences between beginning and experienced teachers--their ways of looking at and responding to textbook material--and two areas where the similarities were more striking than the differences--their notions about representing subtraction with regrouping and their understanding of the topic.

One significant difference between the two groups of teachers concerned how they viewed the textbooks. The experienced teachers used their teaching experience as a lens for looking at the textbooks. Past teaching experience provided them with a sense of the topic, the pedagogical sequence for helping students to learn it (e.g., What else students would have to learn before they came to this?), and the kinds of representations useful in teaching it. As the experienced teachers fit the textbooks into their curriculum scripts, they then moved back and forth between the two selections in ways that matched how they thought the topic should be taught. In contrast, the beginning teachers' ideas were much more diffuse. They seemed to have pieces of ideas about the topic and how to teach it, and they responded to the textbook sections not through a single lens but in terms of these pieces (e.g., the role of real-life examples to personalize the mathematics, the need for manipulatives). More than the experienced teachers, beginners seemed to be stimulated by what they saw in the textbooks themselves.

In contrast to this difference in their ways of looking at the textbooks, experienced and beginning teachers all talked about manipulatives. They seemed to share an assumption that using manipulatives was inherently worthwhile. Neither beginning nor experienced teachers, however, discriminated significantly among particular concrete materials and all seemed to think that if children can see something concrete or can touch and manipulate materials, they will automatically understand. Finally, the beginning and experienced teachers did not differ significantly in their own understandings of multidigit subtraction. Both groups reflected the same range of understandings. Some teachers construed the topic as primarily

about subtraction. Other teachers emphasized the "regrouping" aspect, but even here, their language was tinged with the mechanics of the procedure.

Common belief holds that teachers learn about their subjects by teaching them (Ball and McDiarmid, in press). If that were true, we would have expected the experienced teachers to articulate a deeper understanding of the mathematics. Instead we found that the experienced teachers were no more focused on the conceptual underpinnings of the "borrowing" algorithm than the beginning teachers. This raises questions about the role of experience in learning to teach. Can teaching experience guarantee a deeper understanding of mathematical content? Can it lead to a more refined appreciation of the uses and limits of different concrete materials? Can it yield knowledge of how children learn particular topics? What might cause them difficulty? What might they already understand or find easy?

Subject-specific knowledge of children's learning seems a more likely outcome of teaching experience than knowledge of the subject itself. Nothing ensures that teaching "borrowing," for example, will help teachers unpack the fundamental concepts that underlie the traditional algorithm for subtraction with regrouping. Without such understandings of the content, teachers are also unlikely to recognize or discover critical differences among alternative concrete representations. In short, both preservice and inservice teacher educators need to pay more attention to helping teachers gain conceptual understanding of their subject matter.

The fact that the beginning and experienced teachers differed in their ways of looking at the text selections does suggest that teaching experience makes an important contribution to teachers' learning. In appraising the curricular materials, experienced teachers had more ideas about teaching the topic and more experience in helping children learn it. Thus their teaching scripts (Putnam, 1987) were more developed than those of the beginning teachers who viewed the excerpts in piecemeal fashion. While experienced teachers had more elaborate scripts, they were not necessarily good scripts. As it was for beginning teachers, a major weakness for experienced teachers was the generic use of manipulatives.

Here textbooks might play an important role in developing teachers' understandings of the appropriate use of concrete materials. A recent edition of a highly regarded elementary mathematics texts, for example, advises teachers to "use base ten materials." This advice will not be helpful to teachers who do not already know about the possible choices of manipulatives and the differences among them and may well contribute to indiscriminate use of such materials. The prevalence of textbooks in elementary mathematics teaching underscores the importance of these issues.

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Appendix A

Mathematics Around Us: Teacher's Edition, Grade 2

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pages 217, 218

Subtraction with renaming

objectives

- Rename numbers so that there are more than 9 ones.
- Find the difference of two numbers less than 100.

pre-book activities

1. Put exercises similar to the following on the board.

tens	ones	tens	ones
9	5	5	7
- 2	7	- 1	4

Have the children tell how they would find each difference. Then have them copy and complete each exercise. Ask various children to give the missing numbers for each exercise. Use the results of this activity to determine which children need more help before they do the work on pages 217-218.

2. For those children who need help, adapt and use the suggestions given for pages 215-216.

use of the pages

page 217 Before you have the children work the exercises independently, you may want to talk with the children about school fairs. Ask if any of them have been to a school fair, what kind of prizes they won, what kind of food they bought, and what other things they bought. After you read the directions, point out that the object at the top of each box shows the kind of object (or names the child), that the top number in each exercise tells how many things (or how much money) there were to begin with, and that the bottom number tells how many were sold (or how much was spent). After the children have found each difference, ask questions about each row. For example, for the bottom row, you might ask, "Who had the most money left? Who had the least? Who had more than 30¢ left? Who had less than 30¢?"

Downer Street School had a fair.
How much food was not sold?

tens	ones	tens	ones	tens	ones	tens	ones
5	14	8	11	6	5	5	10
- 4	6	- 7	9	6	0	5	5
18		12		5		5	

How many prizes were left?

tens	ones	tens	ones	tens	ones	tens	ones
3	13	3	11	4	10	1	15
- 4	9	- 1	4	- 5	6	- 1	8
34		27		44		7	

tens	ones	tens	ones	tens	ones	tens	ones
3	10	5	11	5	13	7	17
- 2	2	- 2	6	- 3	4	- 4	8
18		35		29		39	

(seen) 217

page 218 Have the children complete the exercises in the first row. When all are done, ask them what they noticed about the differences in this row (three of the differences are 23). Have the children circle, or mark in some other way, each exercise that has an answer of 23. Tell them that they are to work the exercises in each row and then circle the exercises that have the same difference.

post-book activities

1. Use any of the appropriate post-book activities suggested for pages 215-216.

2. Give each child the worksheet suggested in post-book activity 1 for pages 215-216, or use Teacher Aid 14. Tell a number story and have the children write the appropriate numerals in the box you designate and find the answer. Limit your stories to those in which the children must subtract to find the answers. The following are examples of stories you might use.
 - 42 apples. 16 were sold. How many apples were left?
 - 68 trucks. 35 buses. How many more trucks?
 - 59 boys. 83 girls. How many more girls?
 - 91 geese. 27 flew away. How many geese were left?

Subtract. In each row, mark the answers that are the same.

$\begin{array}{r} \text{tens ones} \\ 8 \cancel{0} \\ -5 \ 7 \\ \hline 23 \end{array}$	$\begin{array}{r} \text{tens ones} \\ 3 \ 9 \\ -1 \ 6 \\ \hline 23 \end{array}$	$\begin{array}{r} \text{tens ones} \\ 4 \ 12 \\ 8 \ 2 \\ -2 \ 8 \\ \hline 24 \end{array}$	$\begin{array}{r} \text{tens ones} \\ 9 \ 12 \\ 8 \ 2 \\ -5 \ 9 \\ \hline 23 \end{array}$
$\begin{array}{r} \text{tens ones} \\ 7 \ 9 \\ -1 \ 8 \\ \hline 61 \end{array}$	$\begin{array}{r} \text{tens ones} \\ 8 \ 14 \\ 8 \ 4 \\ -3 \ 9 \\ \hline 45 \end{array}$	$\begin{array}{r} \text{tens ones} \\ 8 \ 12 \\ 8 \ 2 \\ -4 \ 7 \\ \hline 45 \end{array}$	$\begin{array}{r} \text{tens ones} \\ 7 \ 6 \\ -3 \ 1 \\ \hline 45 \end{array}$
$\begin{array}{r} \text{tens ones} \\ 4 \ 16 \\ 8 \ 0 \\ -1 \ 3 \\ \hline 37 \end{array}$	$\begin{array}{r} \text{tens ones} \\ 3 \ 12 \\ 4 \ 2 \\ - \ 3 \\ \hline 39 \end{array}$	$\begin{array}{r} \text{tens ones} \\ 9 \ 16 \\ 8 \ 8 \\ -6 \ 9 \\ \hline 27 \end{array}$	$\begin{array}{r} \text{tens ones} \\ 8 \ 14 \\ 8 \ 4 \\ -2 \ 7 \\ \hline 37 \end{array}$
$\begin{array}{r} \text{tens ones} \\ 1 \ 10 \\ 2 \ 0 \\ - \ 4 \\ \hline 16 \end{array}$	$\begin{array}{r} \text{tens ones} \\ 2 \ 14 \\ 3 \ 4 \\ -1 \ 8 \\ \hline 16 \end{array}$	$\begin{array}{r} \text{tens ones} \\ 6 \ 18 \\ 7 \ 8 \\ -5 \ 9 \\ \hline 16 \end{array}$	$\begin{array}{r} \text{tens ones} \\ 4 \ 18 \\ 8 \ 8 \\ -3 \ 7 \\ \hline 16 \end{array}$
$\begin{array}{r} \text{tens ones} \\ 8 \ 18 \\ 8 \ 8 \\ -3 \ 5 \\ \hline 58 \end{array}$	$\begin{array}{r} \text{tens ones} \\ 8 \ 11 \\ 8 \ 1 \\ - \ 3 \\ \hline 58 \end{array}$	$\begin{array}{r} \text{tens ones} \\ 6 \ 18 \\ 7 \ 8 \\ -2 \ 7 \\ \hline 48 \end{array}$	$\begin{array}{r} \text{tens ones} \\ 7 \ 11 \\ 8 \ 1 \\ -2 \ 7 \\ \hline 54 \end{array}$

218 (two hundred eighteen)

Subtraction with renaming

Some of the abler children may enjoy suggesting stories for the other children to solve.

3. Practice pages 82 and 83

helping the low achiever

The difficulty that some children have with subtraction computation may be attributed to a weakness in subtraction basic facts or to a failure to understand the idea of regrouping. Before you provide any reteaching for these children, try to determine the cause of their difficulties.

If the difficulty can be traced to a weakness in subtraction basic facts, provide activities such as those given

for pages 211-212. If the difficulty is caused by a failure to understand the idea of regrouping, use activities similar to those suggested for pages 171-174.

providing for the high achiever

1. Give each child a copy of the worksheet suggested in post-book activity 1, pages 215-216, or use Teacher Aid 14. Have these children give the numbers for the exercises in a row. At least two exercises in each row should have the same answer.

After they have completed their work, have various children, in turn, put their exercises on the board. Have the rest of the class copy the exercises, find the answers, and mark the ex-

ercises that have the same answers. The child who suggested the exercises should be the judge.

2. Individualizing page H33

Appendix B

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48

Subtracting 2-Digit Numbers*

Purpose

The purpose of this lesson is to introduce a general procedure for subtracting 2-digit numbers.

This is the third of 8 lessons leading to mastery of the 2-digit subtraction algorithm.

Materials

For demonstration and seminar and Workbook Base-ten materials.

Response cube exercise

(about 3 minutes)

Give practice with the addition and subtraction facts.

Demonstration and seminar

(about 18 minutes)

Subtraction with regrouping This might best be done in 4 steps:

1. Show the children 5 bunches of 10 and 3 more sticks. Ask them how many you have. (53) Ask how many would be left if 25 were taken away. (28) Allow the children to suggest answers and discuss how they got them. Try to take away 25; show that you must take the rubber band off 1 bunch of 10 (if this has not already been pointed out by a child). Write on the board and solve:

$$\begin{array}{r} 4 \text{ } 13 \\ \cancel{8} \cancel{8} \\ - 2 \text{ } 5 \\ \hline (2 \text{ } 8) \end{array}$$

$$\begin{array}{r} 4 \text{ } 13 \\ \cancel{8} \text{ tens and } \cancel{8} \\ - 2 \text{ tens and } 5 \\ \hline (2 \text{ tens and } 8) \end{array}$$

2. Give 53 sticks to one child and 25 to another child. Ask the group who has more. How many more? Discuss. Demonstrate that if the children match their sticks, the second child will run out after a while and the first child will still have some left. Encourage the group to discuss the relationship between the 2 problems.

3. Starting over, tell the group that you have 25 sticks (show them) and you would like to have 53. Ask what you should do. (Get 28 more.)

4. Do more problems of this kind until the children seem to understand the types of situations that lead to subtraction problems and how to get answers (even if very inefficiently).

Workbook

(about 12 minutes)

Do pages 80 and 81. The children may use ice-cream sticks or other base-ten materials to help solve these problems. Before beginning the Workbook exercise, you might want to ask the children if they need help. Those who don't can do the pages independently and go directly to Workshop

$$53 - 25 = \underline{28}$$

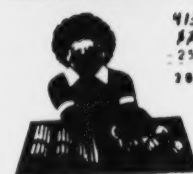
5 tens and 3
- 2 tens and 5



4 tens and 13
- 2 tens and 5



4 tens and 13
- 2 tens and 5
2 tens and 8



80

28

Subtract

$$\begin{array}{r} 27 \\ - 13 \\ \hline 14 \end{array}$$

$$\begin{array}{r} 50 \\ - 25 \\ \hline 25 \end{array}$$

$$\begin{array}{r} 44 \\ - 29 \\ \hline 15 \end{array}$$

$$\begin{array}{r} 83 \\ - 73 \\ \hline 10 \end{array}$$

$$\begin{array}{r} 54 \\ - 28 \\ \hline 26 \end{array}$$

$$\begin{array}{r} 18 \\ - 9 \\ \hline 9 \end{array}$$

$$\begin{array}{r} 26 \\ - 12 \\ \hline 14 \end{array}$$

$$\begin{array}{r} 100 \\ - 25 \\ \hline 75 \end{array}$$

$$\begin{array}{r} 63 \\ - 37 \\ \hline 26 \end{array}$$

$$\begin{array}{r} 52 \\ - 22 \\ \hline 30 \end{array}$$

$$\begin{array}{r} 66 \\ - 57 \\ \hline 9 \end{array}$$

$$\begin{array}{r} 17 \\ - 17 \\ \hline 0 \end{array}$$

81

when they finish. Those who need help can do the pages with you. (Some children may be able to work independently after doing only 1 or 2 problems with you.)

Workshop

(about 12 minutes)

Continue playing the Three Cube Subtraction Game, introduced in Lesson 47.

Extra teaching and practice

See the suggestions given in the Extra Teaching and Practice section of Lesson 47.

Those children who need extra practice may benefit also from taking home directions for the Three Cube Subtraction Game, together with the appropriate response cubes, and playing the game with parents or older sisters or brothers. You might also preteach the Four Cube Subtraction Game, which will be introduced in Lesson 49, and have students take it home also.

Looking ahead

Lesson 50 gives suggestions for group activities that provide subtraction practice in realistic settings. You would do well to read the lesson plan beforehand in order to decide which of the activities you want to undertake and how much preparation time will be necessary.

29